

# Spin Hall effect and Berry phase in two-dimensional electron gas

Shun-Qing Shen

*Department of Physics, The University of Hong Kong, Pokfulam Road, Hong Kong, China*

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The spin Hall effect is investigated in a high mobility two-dimensional electron system with the spin-orbital coupling of both the Rashba and the Dresselhaus types. A spin current perpendicular to the electric field is generated by either the Rashba or the Dresselhaus coupling. The spin Hall conductance is independent of the strength of the coupling, but its sign is determined by the relative ratio of the two couplings. The direction of spin current is controllable by tuning the magnitude of the surface electric field perpendicular to the two-dimensional plane via adjusting the Rashba coupling. It is observed that the spin Hall conductance has a close relation to the Berry phase of conduction electrons.

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## I. INTRODUCTION

Spintronics in semiconductor physics has become an emerging field of condensed matter because of its potential application in information industry, and also because of many essential questions on fundamental physics of electron spin.<sup>1,2</sup> It is tempting to use spin rather than charge of electron for information precessing and storage. Recently a surprising effect was predicted theoretically that an electric field can generate a dissipationless quantum spin current perpendicular to the charge current at room temperatures in conventional hole-doped semiconductors such as Si, Ge, and GaAs.<sup>3</sup> The effect was found to be intrinsic in electron systems with substantial spin-orbit coupling, and the spin Hall conductance has a universal value.<sup>4</sup> Based on this effect it is possible to realize spin injection in paramagnetic semiconductors rather than from the spin polarized carriers in ferromagnetic metals,<sup>5</sup> which is thought as a key step to realize practical spintronic devices. The spin Hall effect due to magnetic impurities was discussed extensively.<sup>6-9</sup> When a charge current circulates in a paramagnetic metal a transverse spin imbalance will be generated, which gives rise to a spin Hall voltage or a spin current.

In two-dimensional (2D) semiconductor heterostructures the spin-orbit interaction can be described in terms of two dominant contributions to the model Hamiltonian, which are directly related to symmetry of the low dimensional geometry. One type is Rashba coupling stemming from the structure inversion asymmetry of confining potential,<sup>10</sup>

$$H_R = -\lambda \boldsymbol{\sigma} \cdot (\mathbf{z} \times \mathbf{k}) = -\lambda(k_x \sigma_y - k_y \sigma_x), \quad (1)$$

where  $\sigma_a$  ( $a=x, y, z$ ) are the Pauli matrices and  $k_x$  and  $k_y$  are two components of the wave vector. The strength of the coupling  $\lambda$  can be modified by a gate field up to 50%,<sup>11</sup> such that it can be applied to control spin transport, such as in spin field transistor.<sup>12,13</sup> Another type is Dresselhaus coupling from the bulk inversion asymmetry,<sup>14</sup>

$$H_D = -\beta(k_x \sigma_x - k_y \sigma_y). \quad (2)$$

In some materials such as GaAs the two types of spin-orbit coupling are usually of the same order of magnitudes. The interplay of them has been investigated theoretically with

respect to several phenomena, such as the nonballistic spin field effect transistor<sup>15</sup> and electron-spin manipulation.<sup>16</sup>

In this communication we investigate the spin Hall effect in a 2D electron system with spin-orbit coupling of both Rashba and Dresselhaus types. Consider the Hamiltonian,

$$H_0 = \frac{\hbar^2}{2m} \mathbf{k}^2 + H_R + H_D. \quad (3)$$

We find the Berry phase in the eigenstates to be 0 for  $|\lambda| = |\beta|$ ,  $+\pi$  for  $|\lambda| > |\beta|$ , and  $-\pi$  for  $|\lambda| < |\beta|$ . When the system is subjected to an electric field in the 2D plane the spin-orbit coupling leads to a dissipationless spin current perpendicular to the electric field, and polarized in the direction perpendicular to the 2D plane when  $|\lambda| \neq |\beta|$ . The spin Hall conductivity has a universal value except that its sign is determined by the relative ratio of two couplings or the sign of the Berry phase. As Rashba coupling is tunable by a gate field perpendicular to the 2D plane it is possible to control the direction of spin current by adjusting the magnitude rather than direction of the gate field when the system is near  $|\lambda| = |\beta|$ .

## II. EIGENSTATES OF $H_0$ AND THE BERRY PHASE

The system with two types of coupling has been investigated by several authors. The Hamiltonian can be diagonalized exactly. The eigenstates are

$$|k_+, +, \theta\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta} \\ +i \end{pmatrix} \otimes |k_+\rangle, \quad (4a)$$

$$|k_-, -, \theta\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta} \\ -i \end{pmatrix} \otimes |k_-\rangle. \quad (4b)$$

where  $|k\rangle$  is the eigenket of  $k$ , and  $\theta$  is given by

$$\tan \theta = \frac{\lambda k_y - \beta k_x}{\lambda k_x - \beta k_y} = \frac{\lambda \sin \varphi - \beta \cos \varphi}{\lambda \cos \varphi - \beta \sin \varphi} \quad (5)$$

with  $k_x = k \cos \varphi$  and  $k_y = k \sin \varphi$ . We choose  $k_{\pm}$  such that two eigenstates are degenerated

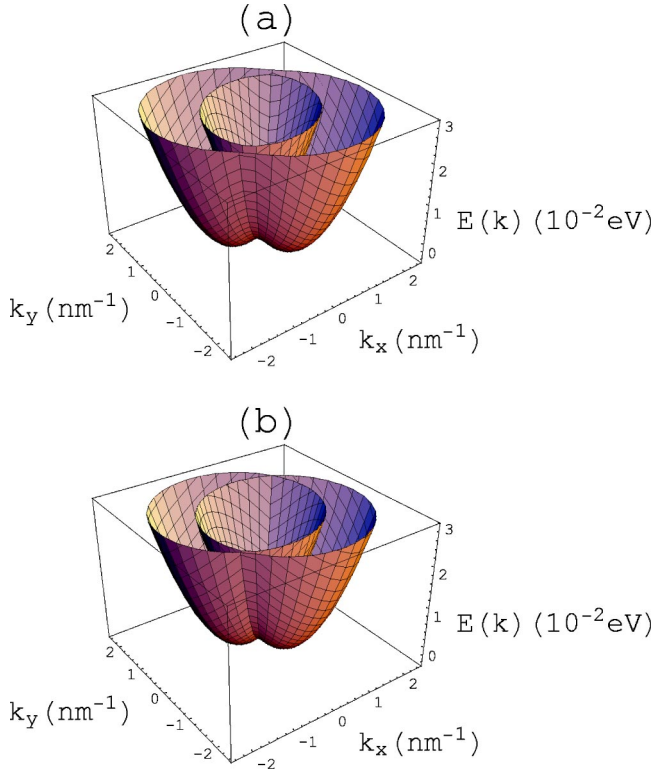


FIG. 1. (Color online) Two energy bands: the outer shell is for  $k_+$  band, and the inner shell is for  $k_-$  band. The particle mass  $m$  is 0.067 in units of the bare electron mass  $m_0$ .  $k$  is in units of  $\text{nm}^{-1}$ . (a)  $\lambda = 5.0 \times 10^{-3} \text{ eV nm}$  and  $\beta = 7.5 \times 10^{-3} \text{ eV nm}$  or  $\lambda = 7.5 \times 10^{-3} \text{ eV nm}$  and  $\beta = 5.0 \times 10^{-3} \text{ eV nm}$ . The two cases give the same shape of energy spectra, but the corresponding spin states are different by  $\theta \rightarrow \pi/2 - \theta$ . The two bands coincide at the original point. (b)  $\lambda = \beta = 5.0 \text{ meV nm}$ . The two bands cross over at  $k_x = k_y$  or  $\varphi = \pi/4, 5\pi/4$ .

$$E(k) = \frac{\hbar^2}{2m} k_+^2 - k_+ \Delta\omega(\varphi) = \frac{\hbar^2}{2m} k_-^2 + k_- \Delta\omega(\varphi) \quad (6)$$

with  $\Delta\omega(\varphi) = \sqrt{\lambda^2 + \beta^2 - 2\lambda\beta \sin 2\varphi}$ . In general the two bands do not cross over except at  $k=0$ . In the case of  $\lambda = \pm\beta$ ,  $\theta = -\pi/4$  if  $\lambda = \beta$  and  $\pi/4$  if  $\lambda = -\beta$ . The spin states in Eq. (4) are independent of  $k$  and  $\varphi$ . Furthermore  $k\Delta\omega=0$  when  $k_x = k_y$  or  $\varphi = \pi/4$ . It is noted that there exists an additional conserved quantity,  $(\sigma_x \pm \sigma_y)/\sqrt{2}$ .<sup>15</sup> The difference of two  $k_{\pm}$  is given by

$$k_+(\varphi) - k_-(\varphi) = \frac{2m}{\hbar^2} \Delta\omega(\varphi), \quad (7)$$

which is independent of  $k$ . The two energy bands are plotted in Fig. 1. In these states the Berry phase,<sup>17</sup> which is acquired by a state upon being transported around a loop in the  $k$  space, can be evaluated exactly,

$$\gamma_{\pm} = \oint dl \cdot \langle k_{\pm}, \pm, \theta | i \frac{\partial}{\partial \mathbf{k}} | k_{\pm}, \pm, \theta \rangle = \frac{\lambda^2 - \beta^2}{|\lambda^2 - \beta^2|} \pi \quad (8)$$

if  $|\lambda| \neq |\beta|$ , and is equal to zero if  $|\lambda| = |\beta|$ . The disappearance of the Berry phase at  $|\lambda| = |\beta|$  may be relevant to the cross-

over of two energy bands. We notice that if we make a replacement,  $\lambda \rightarrow \beta$  and  $\beta \rightarrow \lambda$  the angle  $\theta \rightarrow \pi/2 - \theta$ . This property gives different Berry phases for  $|\lambda| > |\beta|$  and  $|\lambda| < |\beta|$ . According to the theory of Sundaram and Niu,<sup>18</sup> the Berry phase is closely related to the electron transport in semiconductors. It is found that the Berry phase and magnetic monopoles in momentum space generate anomalous Hall effect in some ferromagnetic materials.<sup>19</sup>

### III. QUANTUM DYNAMICS IN AN ELECTRIC FIELD

We shall consider the effect of uniform electric field  $E$ . Our whole Hamiltonian is thus given by  $H = H_0 + eEx$ , assuming that the electric field is along the  $x$  direction. We start with the Heisenberg equation of motion,  $i\hbar(d/dt)O = [O, H]$ . The wave vector is determined by  $k_x(t) = k_{x0} - (eE/\hbar)t$  and  $k_y(t) = k_{y0}$  where  $k_{x0}$  and  $k_{y0}$  are the initial values at  $t=0$ . The set of equations for spin operators are given by

$$\frac{\partial}{\partial t} \sigma_x = -\frac{2}{\hbar} (\lambda k_{x0} - \beta k_{y0}) \sigma_z + \frac{2\lambda eE}{\hbar^2} t \sigma_z, \quad (9a)$$

$$\frac{\partial}{\partial t} \sigma_y = -\frac{2}{\hbar} (\lambda k_{y0} - \beta k_{x0}) \sigma_z - \frac{2\beta eE}{\hbar^2} t \sigma_z, \quad (9b)$$

$$\begin{aligned} \frac{\partial}{\partial t} \sigma_z = & + \frac{2}{\hbar} (\lambda k_{x0} - \beta k_{y0}) \sigma_x + \frac{2}{\hbar} (\lambda k_{y0} - \beta k_{x0}) \sigma_y - \frac{2\lambda eE}{\hbar^2} t \sigma_x \\ & + \frac{2\beta eE}{\hbar^2} t \sigma_y. \end{aligned} \quad (9c)$$

When  $E=0$ , the equations can be solved exactly

$$\begin{aligned} \sigma_x^0(t) = & (\cos^2 \theta \sigma_x(0) + \sin \theta \cos \theta \sigma_y(0)) \cos \omega t \\ & - \cos \theta \sigma_z(0) \sin \omega t + \sin^2 \theta \sigma_x(0) - \sin \theta \cos \theta \sigma_y(0), \end{aligned} \quad (10a)$$

$$\begin{aligned} \sigma_y^0(t) = & (\sin \theta \cos \theta \sigma_x(0) + \sin^2 \theta \sigma_y(0)) \cos \omega t \\ & - \sin \theta \sigma_z(0) \sin \omega t - \sin \theta \cos \theta \sigma_x(0) + \cos^2 \theta \sigma_y(0), \end{aligned} \quad (10b)$$

$$\sigma_z^0(t) = \sigma_z(0) \cos \omega t + (\cos \theta \sigma_x(0) + \sin \theta \sigma_y(0)) \sin \omega t. \quad (10c)$$

The characteristic frequency  $\omega = (2/\hbar)k\Delta\omega(\varphi)$ . Until now we have no general solution for the problem in the presence of an electric field. In this communication we are concerned with the linear response of the transport properties to the field. For our purpose we only need an asymptotic solution for a weak field  $E$  and a short instant  $t$ . We expand the spin operators in terms of  $E$ :  $\sigma_a(t) = \sigma_a^0(t) + \Delta\sigma_a^1(t) + \Delta\sigma_a^2(t) + \dots$ . For a short instant  $t$ , we have an asymptotic solution for the linear correction to  $\sigma_a^0(t)$  due to the electric field,

$$\Delta\sigma_x^1 \approx + \frac{eE}{2k^2} \frac{\lambda \sigma_x^0(t)}{[\Delta\omega(\varphi)]^2}, \quad (11a)$$

$$\Delta\sigma_y^1 \approx -\frac{eE}{2k^2} \frac{\beta\sigma_z^0(t)}{[\Delta\omega(\varphi)]^2}, \quad (11b)$$

$$\Delta\sigma_z^1 \approx -\frac{eE}{2k^2} \frac{\lambda\sigma_x^0(t) - \beta\sigma_y^0(t)}{[\Delta\omega(\varphi)]^2}. \quad (11c)$$

We see that the effective field caused by the spin-orbit coupling has a non-zero correction to the spin operators even when  $t \rightarrow 0$ .

#### IV. SPIN CURRENT AND UNIVERSAL SPIN HALL CONDUCTANCE

Of cause the electric field will induce a charge current in an electron gas. The problem has already been studied extensively.<sup>20</sup> Spin-orbit coupling leads to a non-zero Hall resistance proportional to  $\lambda$  and  $\beta$ ,  $\rho_{xy} \propto \text{sign}(\lambda)|\lambda\beta|$ . In this communication we focus on the spin Hall effect. We notice that the electric field causes a linear correction to spin operators in Eq. (11) when  $t \rightarrow 0$ . This correction will generate the spin Hall effect as discussed by Murakami *et al.*<sup>3</sup> and Sinova *et al.*<sup>4</sup> Spin current, polarized perpendicular to the 2D plane and flowing perpendicular to the electric field,<sup>21</sup> is defined as

$$j_y^z = \frac{\hbar}{4} \left\langle \left\{ \sigma_z(t), \frac{\partial}{\partial t} y(t) \right\} \right\rangle, \quad (12)$$

where  $(\partial/\partial t)y = (\hbar/m)k_{y0} + (\lambda/\hbar)\sigma_x(t) + (\beta/\hbar)\sigma_y(t)$ . The expectation value is taken over the energy eigenstates of electrons. The linear term to the electric field is given by

$$j_y^z = \sum_k \frac{\hbar^2}{2m} \langle k_+, +, \theta | \Delta\sigma_z^1(t) k_{y0} | k_+, +, \theta \rangle n_F[E(k_+) - \mu] \\ + \sum_k \frac{\hbar^2}{2m} \langle k_-, -, \theta | \Delta\sigma_z^1(t) k_{y0} | k_-, -, \theta \rangle n_F[E(k_-) - \mu], \quad (13)$$

where  $n_F[E(k_{\pm}) - \mu]$  is the Dirac-Fermi distribution and  $\mu$  is the chemical potential. At zero temperature we have

$$j_y^z = \frac{\hbar^2 e E}{16\pi^2 m} \int_0^{2\pi} d\varphi \frac{(\lambda^2 - \beta^2) \sin^2 \varphi [k_+ - k_-]}{(\lambda^2 + \beta^2 - 2\lambda\beta \sin 2\varphi)^{3/2}}.$$

By using the relation in Eq. (7) we obtain the spin Hall conductance

$$\sigma_{sH} = j_y^z / E = \frac{e}{8\pi^2} \gamma_{\pm} \quad (14)$$

as long as both bands are occupied.<sup>22</sup> We have ignored the time-dependent terms. It is an interesting observation that the spin Hall conductance is closely relevant to the Berry phase of electron states in the absence of the electric field. It deserves further investigation on the relation in general cases. Spin current caused by the Berry phase should not contribute to dissipation, although a charge current will do. If  $\beta = 0$   $\sigma_{sH} = e/8\pi$  which recovers the result of Sinova *et al.*<sup>4</sup> and followed by several authors more recently.<sup>23,24</sup> If  $\lambda = 0$ , Dresselhaus coupling can also produce a spin Hall conduc-

tance  $\sigma_{sH} = -e/8\pi$ . Very interestingly two spin Hall conductances differ by only a sign. Competition of two couplings also cancels the effect at  $\lambda = \pm\beta$ . It seems that the effect caused by Dresselhaus coupling dominates if  $|\beta| > |\lambda|$  while the effect caused by Rashba coupling dominates if  $|\lambda| > |\beta|$ . Very recently Rashba<sup>25</sup> observed that even in the absence of an external electric field there exists a tiny spin current  $J_y^x = -J_x^y \neq 0$  in the open boundary condition. The type of spin current along spin  $z$  direction does not exist in the absence and presence of an electric field.

To explore the physical origin of the sign change of spin Hall conductance near  $\lambda = \beta$  we find a new symmetry in the system with the two spin-orbital couplings. Performing an unitary transformation,

$$\sigma_x \rightarrow \sigma_y; \sigma_y \rightarrow \sigma_x; \sigma_z \rightarrow -\sigma_z, \quad (15)$$

the two terms of Rashba and Dresselhaus couplings are changed to

$$H_R = -\lambda(k_x\sigma_y - k_y\sigma_x) \rightarrow -\lambda(k_x\sigma_x - k_y\sigma_y), \quad (16a)$$

$$H_D = -\beta(k_x\sigma_x - k_y\sigma_y) \rightarrow -\beta(k_x\sigma_y - k_y\sigma_x). \quad (16b)$$

The spin current along the  $z$  direction changes a minus sign,  $J_y^z \rightarrow -J_y^z$  as  $\sigma_z \rightarrow -\sigma_z$ . Thus this symmetry indicates that the system with Rashba coupling  $\lambda$  and Dresselhaus coupling  $\beta$  and the system with Rashba coupling  $\beta$  and Dresselhaus coupling  $\lambda$  have the same spin Hall conductance (along spin  $z$  direction), but with opposite signs. At the symmetric point  $\lambda = \beta$ , we conclude that  $J_y^z = 0$ , i.e., the spin Hall effect is suppressed completely. This general conclusion is based on symmetry analysis of the system and is mathematically rigorous. It is in agreement with the result of the linear response calculation in Eq. (14).

Now we come to address the effects of a finite electron quasi-particle lifetime and scattering mechanism, which was discussed in a system with Rashba coupling by several authors. Schliemann and Loss<sup>23</sup> discussed the correction of spin conductance due to the effect of finite lifetime  $\tau$  for  $\beta = 0$  using the Kubo formula in the zero frequency limit and concluded that the universal value for spin conductance is valid only when the "Rashba energy"  $\epsilon_R = m\lambda^2/\hbar^2$  should be comparable with the energy scale  $\hbar/\tau$ . Fortunately in some semiconductors such as GaAs quantum wells  $\hbar/\tau < \epsilon_R$  and it is safe to neglect these effects. On the other hand, more recently Burkov and MacDonald found that the spin Hall conductance in a system with Rashba coupling remains the universal value  $e/8\pi$  in the case of strong disordering as in the case of weak disordering.<sup>26</sup> Thus the scattering effect depends on the scattering mechanisms.

#### V. CONTROLLABLE RASHBA COUPLING AND REVERTIBLE SPIN CURRENT

Rashba coupling and Dresselhaus coupling are related to the symmetry of quantum wells. In GaAs quantum wells both terms are usually of the same order of magnitude while the narrow gap compounds like in InAs Rashba coupling dominate.<sup>27</sup> However there is no difficulty to achieve the

situation of  $\lambda = \beta$ . In the case of  $\lambda = \pm\beta$  the Elliot-Yafet spin flip mechanism is suppressed completely since the spin states in  $|k_{\pm}, \pm, \theta = \pm\pi/4\rangle$  are independent of  $\varphi$ . The Berry phase of the states  $|k_{\pm}, \pm, \theta\rangle$  becomes zero and correspondingly there is no spin Hall effect. Thus the electric field cannot generate a dissipationless spin current. If  $|\lambda| > |\beta|$   $\sigma_{SH} = e/8\pi$  while  $\sigma_{SH} = -e/8\pi$  if  $|\lambda| < |\beta|$ . The spin current changes a sign (or direction). As Rashba coupling is adjustable by a gate field perpendicular to the electron gas plane<sup>11</sup> we may change  $\gamma$  around  $\beta$ ,  $\gamma = \beta + \delta\Delta E_{\perp}$ , by adjusting the perpendicular electric field  $\Delta E_{\perp}$ . Thus by using the property of sign change in the Berry phase we can realize to revert the spin current by adjusting the magnitude rather than the direction of the gate field.

## VI. CONCLUSION

The spin Hall conductance caused by Dresselhaus coupling has the same universal value, but an opposite sign to

that caused by Rashba coupling. It is an interesting observation that the spin Hall conductance has a close relation to the Berry phase of single electron states. The spin current in this effect can be revertible when the amplitudes of two couplings are comparable by adjusting the gate voltage via Rashba coupling. It is anticipated that this effect will be applicable to the spintronic devices in the future.

Recently, the author became aware that Sinova *et al.* arrived at the same spin Hall conductances by means of the linear response theory independently.<sup>28</sup>

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- <sup>21</sup>Due to the anomalous Hall effect, a charge current along the  $y$  direction as well as along the  $x$  direction will be induced,  $j_x = \sigma_{xx}E_x$  and  $j_y = \sigma_{yx}E_x$ . Thus the total electric current is not parallel to the electric field.
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